CONSTRUCTION OF THE ENTROPY SOLUTION OF HYPERBOLIC CONSERVATION LAWS BY A GEOMETRICAL INTERPRETATION OF THE CONSERVATION PRINCIPLE. H. Boing, K. Werner, and H. Jackisch, *Institut für Geometrie und Praktische Mathematik, RWTH Aachen, 5100 Aachen, WEST GERMANY (FRG).*

In this paper we consider scalar hyperbolic equations in one space dimension of the type

$$u_t(x, t)+\frac{d}{dx}f(u; x)=h(u; x), \qquad u(x, 0)=u_0(x), \quad x\in\mathbb{R}, \quad t>0, \tag{1}$$

where $f\in C^1$ and h is continuous w.r.t. u; x. The initial condition is assumed to be piecewise continuous. We present a new method for constructing the entropy solution of (1) at a fixed time $t=T>0$ in *one* time step based on transporting the initial values along characteristics. If the solution of (1) is smooth, we obtain the exact solution; in case of shocks the multivalued graph of the initial data is corrected by a geometrical averaging technique via the conservation principle. The method is also applicable to a scalar equation in which there is a mild coupling between the physical dimensions in the problem, for example,

$$u_t(x, y)+\frac{d}{dx}f(u; x, y)+\frac{d}{dy}f(u; x, y)=h(u; x, y). \tag{2}$$

By a change of variables, (2) can be reduced to a quasi one-dimensional problem. We conjecture that the advantage of computing the entropy solution at a fixed time in one time step cannot easily be carried over to systems. But we have some hints that this might be possible in case of scalar equations in two space dimensions with arbitrary fluxes f_1, f_2. The CPU time depends only on the total number of shocks which occur in the entropy solution up to time T; the accuracy of the computed shock position is of order at least 10^{-2}. Since our method is not based on a time discretisation, questions (and problems) concerning stability and convergence do not arise.

ADAPTIVE GRID GENERATION FROM HARMONIC MAPS ON RIEMANNIAN MANIFOLDS. Arkady S. Dvinsky, *Creare Inc., Hanover, New Hampshire, USA.*

In this paper we describe a new method for generation of solution adaptive grids based on harmonic maps on Riemannian manifolds. The reliability of the method is assured by an existence and uniqueness theorem for one-to-one maps between multidimensional multiconnected domains. We formulate an adaptive Riemannian metric consistent with this theorem. Several examples demonstrating application of the developed procedure are provided.

DIFFUSING-VORTEX NUMERICAL SCHEME FOR SOLVING INCOMPRESSIBLE NAVIER–STOKES EQUATIONS. Zhi Yun Lu, *New York Institute of Technology, Old Westbury, New York, USA*; Timothy J. Ross, *University of New Mexico, Albuquerque, New Mexico, USA.*

A new numerical algorithm, the diffusing-vortex method for time-dependent two-dimensional Navier–Stokes equations, which was previously presented and applied to the incompressible viscous flow past a circular cylinder with high Reynolds number by Lu and Shen, is further developed for extension to general two-dimensional initial value problems and boundary value problems. The new algorithm consists of two steps in a simulation cycle: a Lagrangian convection simulation for the first time step and a diffusion simulation through the use of new vortex points at fixed Eulerian mesh points for the second time step. The mathematical mechanisms of computation behind this algorithm and its characteristics of convergence and accuracy are analyzed in applications for the following problems: (1) an initial value problem involving the decay of a single vortex of finite size and the decay and interaction of a vortex

pair of two finite-core regions; (2) a boundary value problem for the unsteady flow field around a rotating cylinder with high Reynolds number up to $Re_d = U \cdot D/\nu = 10{,}000$. Numerical results are compared with either exact solutions or other numerical methods. The numerical advantages of the diffusing-vortex scheme over other conventional vortex methods, cloud-in-cell methods, particle methods, and some finite difference schemes are evaluated in terms of reducing total CPU time, avoiding cutoff procedures and sidestepping various interpolations.

SPECTRAL COLLOCATION METHODS AND POLAR COORDINATE SINGULARITIES. Henner Eisen, Wilhelm Heinrichs, and Kristian Witsch, *Heinrich-Heine-Universität Düsseldorf, Lehrstuhl für Angewandte Mathematik, Düsseldorf, WEST GERMANY (FRG).*

This paper considers the numerical solution of elliptic differential equations on the unit disk. Using polar coordinates, the disk is mapped onto a rectangle. The resulting transformed problem is solved by a method related to collocation. Since the origin is a coordinate singularity, some natural trial functions are singular there and a special technique is applied to use zero as a collocation point. For Poisson and Helmholtz equations, a fast algorithm with an operation count of $\mathcal{O}(N^2 \log N)$ is presented. Numerical results show the different stability and convergence properties of the algorithms.

A NUMERICAL SCHEME FOR THE SOLUTION OF THE SPACE CHARGE PROBLEM ON A MULTIPLY CONNECTED REGION. C. J. Budd, *Oxford University Computing Laboratory, Oxford, UNITED KINGDOM*; A. A. Wheeler, *School of Mathematics, Bristol, UNITED KINGDOM.*

In this paper we extend the work of Budd and Wheeler, who described a new numerical scheme for the solution of the space charge equation on a simply connected domain, to multiply connected regions. The space charge equation, $\nabla \cdot (\Delta\bar{\varphi} \nabla \bar{\varphi}) = 0$, is a third-order nonlinear partial differential equation for the electric potential $\bar{\varphi}$, which models the electric field in the vicinity of a coronating conductor. Budd and Wheeler described a new way of analysing this equation by constructing an orthogonal coordinate system $(\bar{\varphi}, \bar{\psi})$ and recasting the equation in terms of x, y and $\Delta\bar{\varphi}$ as functions of $(\bar{\varphi}, \bar{\psi})$. This transformation is singular on multiply connected regions and in this paper we show how this may be overcome to provide an efficient numerical scheme for the solution of the space charge equation. This scheme also provides a new method for the solution of Laplaces equation and the calculation of orthogonal meshes on multiply connected regions.

DIRECT SIMULATIONS OF TURBULENT FLOW USING FINITE-DIFFERENCE SCHEMES. Man Mohan Rai, *NASA Ames Research Center, Moffett Field, California, USA*; Parvis Moin, *Stanford University, Stanford, California and NASA Ames Research Center, Moffett Field, California, USA.*

This paper presents finite-difference solutions to the evolution of small-amplitude disturbances and incompressible fully developed turbulent channel flow. The main objective of the paper is to provide a comprehensive comparison between the results obtained using finite-difference and spectral methods. An advantage of finite-difference schemes over the highly accurate spectral methods lies in the ease with which they can be applied to complex geometries. The finite-difference methods used include a kinetic-energy-conserving type of central difference scheme and a high-order-accurate upwind difference scheme. Unlike the central difference scheme, the upwind difference scheme was found not to require a kinetic energy conservation property to control aliasing error. The dissipative nature of the upwind scheme results in a damping of the higher frequency content. As a result very little energy is aliased back. The computed data (including first- and second-order statistics) for the turbulent channel flow case are found to compare well with experimental data and earlier spectral simulations. It appears that the high-order-accurate upwind scheme is a good candidate for direct simulations of turbulent flows over complex geometries.